MODELING WARM-UP DRIFT IN COMMERCIAL HARMONIC PHASE STANDARDS[*]

Jeffrey A. Jargon, Jolene D. Splett, Dominic F. Vecchia, and Donald C. DeGroot

National Institute of Standards and Technology
325 Broadway, M/S 818.01, Boulder, CO 80305, USA

Abstract

We develop an empirical model for the warm-up drift in harmonic phase standards used to calibrate the phase distortion of nonlinear vector network analyzers. This model will enable us to estimate the time at which the standards reach stability.

Introduction

A class of instruments known as nonlinear vector network analyzers (NVNA) are capable of characterizing nonlinear devices under realistic, large-signal operating conditions [1]. To do this, complex traveling waves are measured at the ports of a device both at the stimulus frequency (or frequencies), and at other frequencies that are part of the large-signal response. These include harmonics and intermodulation products created by the nonlinearity of the device, in conjunction with impedance mismatches between the system and the device. The calibration of a commercial NVNA consists of three steps: a relative calibration identical to that used in a linear vector network analyzer, an amplitude calibration that makes use of a power meter, and a phase distortion calibration that makes use of a harmonic phase standard. All are performed on a frequency grid related to the source tones and the anticipated nonlinear response of the device.

A commercial harmonic phase standard (HPS) is driven at a fundamental frequency and produces a harmonic-series output signal. The HPS, which is used as a transfer standard, is characterized by a sampling oscilloscope, which in turn is characterized by a nose-to-nose calibration [2]. In this way, we transfer the phase-dispersion calibration of an oscilloscope to "knowing" the phase relationship of each harmonic of the HPS.

In a previous study [3], we presented a repeatability study of two commercial harmonic phase standards measured by an NVNA. By performing multiple calibrations and measurements, we determined the repeatability bounds for the phases and magnitudes of each harmonic component by utilizing the propagation-of-errors method to compute expanded uncertainties. We also studied the possibility of warm-up drift in the two devices, and discovered considerable drift as a function of time, with an estimated $1/e$ time-constant of around 500 seconds, which is much longer than the warm-up time of 120 seconds set by the manufacturer's control software.

In this paper, we develop an empirical model for the warm-up drift of HPS devices, which will enable us to estimate their phase angle response stability time-points.

Drift Model of the Phase Angle Response

In the process of searching for a suitable empirical model, we found that two first-order decay terms with an intercept produced excellent fits to all drift data collected to date in repeated calibrations runs on both HPS devices. The nonlinear decay model for drifting phase measurements of a given harmonic is

$$p = \phi + \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} + \varepsilon, \tag{1}$$

where p is the measured phase angle, ϕ is the stable phase value after warm-up, $t > 0$ is the time from start-up, α_1 and β_1 are the unknown parameters of the first decay term, α_2 and β_2 are the unknown parameters of the second decay term, and ε is a random error term with mean zero and a standard deviation σ. It is not too surprising that this particular empirical model works so well, considering that each HPS contains two nonlinear components, namely an amplifier and a step-recovery diode.

If the subscript i ($i = 1, \dots n$) denotes the number of repeated measurements and t_i and p_i represent the associated time and measured phase, then estimates of the unknown model parameters $\{\phi, \alpha_1, \beta_1, \alpha_2, \beta_2\}$ can be obtained from the usual nonlinear least-squares solution that minimizes the error sum of squares:

$$S\left(\hat{\phi}, \hat{\alpha}_1, \hat{\beta}_1, \hat{\alpha}_2, \hat{\beta}_2\right) = \sum_{i=1}^{n} (p_i - \hat{p}_i)^2, \tag{2}$$

where $\hat{p}_i$ is the predicted value of the ith measured phase angle, and is given as

$$\hat{p}_i = \hat{\phi} + \hat{\alpha}_1 e^{-\hat{\beta}_1 t} + \hat{\alpha}_2 e^{-\hat{\beta}_2 t}. \tag{3}$$

Here we have used the convention of denoting the least-squares estimates of the parameters by placing a caret over the respective symbols for the unknown parameters. If we assume that the random errors ε_i are independent Gaussian variates, then the least-squares estimates are also the maximum likelihood estimates of the parameters.

Given the least-squares solution, the variance of the random error ε in eq. (1) is estimated, in the usual way, from the residual sum of squares as

$$\hat{\sigma}^2 = \sum_{i=1}^{n} (p_i - \hat{p}_i)^2 / (n - 5). \tag{4}$$

Figure 1 shows the actual measured phase angles of the fifth harmonic together with the estimated curve using the exponential decay model of eq. (1). In this particular experiment, we made 1000 repeated measurements of the 20 GHz HPS using a fundamental frequency of 600 MHz, with a five-second pause between each measurement. The residuals from the fitted curve are shown in Figure 2. The apparent randomness and homogeneity of variance in time are consistent with the assumptions that were made with respect to additive noise in the model.
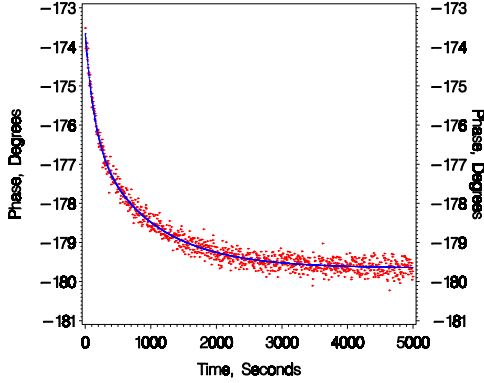

Figure 1. Phase angles of the fifth harmonic of the 20 GHz HPS along with the estimated curve using the exponential decay model.
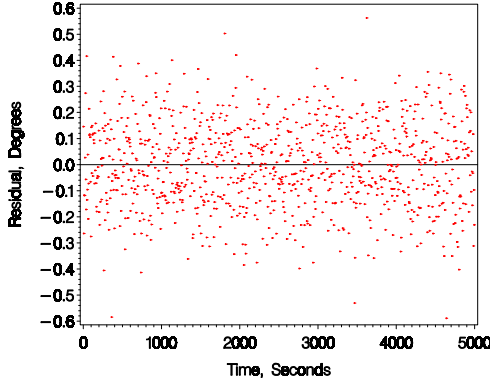

Figure 2. Residuals from the fitted curve.

Estimation of Stable Phase Angle Time-Point

When a phase measurement is made that is consistent with the decay model of eq. (1), the measured phase differs from the stable value ϕ by a systematic error that depends on time, plus a random effect. The actual (unknown) systematic error of a measurement at time t, denoted as $\delta(t)$, is

$$\delta(t) = E(p \mid t) - \phi = \alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t}, \qquad (5)$$

where $E(p|t)$ is the expected phase angle at time t. Since the error $\delta(t)$ is decreasing in time, we define the "true" stable time-point to be the time t when $\delta(t) = \Delta$, where Δ is a given error bound judged to be acceptable with

respect to the intended use of a phase-angle measurement. Since $\delta(t)$ depends on unknown parameters, statistical methods are required to estimate the stable time-point.

Given estimates of the decay parameters from a warm-up experiment, an appropriate estimate of $\delta(t)$ is

$$\hat{\delta}(t) = \hat{\alpha}_1 e^{-\hat{\beta}_1 t} + \hat{\alpha}_2 e^{-\hat{\beta}_2 t}. \qquad (6)$$

The least-squares analysis also provides estimated variances and covariances of the decay parameter estimates, which can be used to derive an approximate upper confidence bound on $\delta(t)$, based on the classical large-sample Normal distribution theory of nonlinear least squares. With this approach, the estimated variance of $\delta(t)$ is obtained by substituting estimated model coefficients, variances and covariances into the propagation-of-errors (POE) formula for the variance of $\hat{\delta}(t)$. Letting $S_{\hat{\delta}(t)} (t \geq 0)$ denote the estimated POE standard error of $\hat{\delta}(t)$, an upper $(100 \times \gamma)$ percent confidence bound $U(t)$ on $\hat{\delta}(t)$ is given by

$$U(t) = \hat{\delta}(t) + N_\gamma S_{\hat{\delta}(t)}, \qquad (7)$$

where N_γ denotes the $100 \times \gamma$ th percentile of the standard Gaussian distribution, and t is the estimated time when $\delta(t) = \Delta$ can be taken as the solution of the equation $U(t) = \Delta$.

The procedure outlined above is useful for retrospective analysis of a single drift experiment or a combined analysis of two or more runs where the drift parameters are believed to be constant regardless of the measurement occasion. However, since the drift parameters may vary significantly from run to run, two other statistical procedures are being developed: (1) real-time "stability" decisions (perhaps to be embedded in system software) or, (2) estimation of a fixed, minimum warm-up time for a particular HPS when sufficient repeat runs are available for analysis. The latter approach accounts for run-to-run differences by treating decay parameters in eq. (1) as random, rather than fixed coefficients. Any procedure would have to simultaneously account for the drift of all relevant harmonics.

References

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